MULTI-CRITERIA PROJECT PORTFOLIO OPTIMIZATION UNDER RISK AND SPECIFIC LIMITATIONS

Jiří Fotr, Miroslav Plevný, Lenka Švecová, Emil Vacík

Introduction

In reality we can often come across the task of developing a portfolio (set) of projects having the character of **investment projects** (extending production capacity by means of development or acquisitions, innovation of production technologies, introducing new products and such like) or **research projects** (research and development of new products, technologies, processes and such like). Creating these portfolios for the individual groups of projects represents a base for conceiving an investment programme or a research programme of a company. Creating portfolios of projects has its own **process aspects** as well as **model aspects**. The process aspects consist in the desired process of the project portfolio development from the point of view of the individual steps and their content [7]. The model aspects, which are dealt with in this contribution, relate to applying certain optimization models supporting the creation of portfolios. With regard to the uncertainty of many factors influencing the future projects results these models are to be perceived as stochastic optimization models.

The aim of the contribution is to briefly characterize the types of the tasks concerning the optimization of a portfolio, to specify the nature of the development of a project portfolio under risk and to show the solution of this task, partly by means of the deterministic equivalents of the stochastic optimization tasks and partly as an application of the optimization tool OptQuest (add-in MS Excel Crystal Ball) for the stochastic optimization. Both these optimization approaches are illustrated on practical examples demonstrating the types of information that can be gained for increasing the quality of investment plans and decision making.

1. Classification of the Portfolio Optimization Tasks

The issue of the development, or, as the case may be, the optimization of a project portfolio under risk belongs to a broad group of the portfolio optimization tasks. This group can be classified according to a number of viewpoints, namely those as follows:

- **the character of variables,** which may be either continuous or discrete. The first works from the field of the portfolio theory dealt with the allocation of financial investment with continuous variables and they are associated with the name of H. Markowitz [23]. Only later attention was paid to the tasks with the discrete or bivalent variables, and these represent the core of this contribution;
- **different level of uncertainty** concerning **parameters** of the portfolio development task, when on the one hand available information makes it possible to reach the determination of their divisions of probability, on the other side it is impossible to determine even the state of the world whereon the values of these parameters depends ([2], [3], [11]);
- **the number of assessment criteria,** when the tasks may be of the **mono-criteria** and **multi-criteria** nature. A larger number of works from the field of portfolio development has a mono-criteria character and these works apply various approaches. These are, for example, the portfolio optimization by means of mathematical fuzzy logic programming [13], the portfolio optimization

with stochastic yields by means of fuzzy information [16], and the choice of a portfolio under stringent uncertainty [3]. As far as multi- criteria oriented works are concerned, it is possible to mention for example the choice of a portfolio by means of multi-criteria stochastic programming [1] or optimization of investment in the selected fields of the national economy based on the method of linking scenarios with the method of the multi-criteria limitation PROMETHEE [15];

- **variables of the model of portfolio development** which have the nature of random quantity. These may become the coefficients of a criteria function, the coefficients of variables under limiting conditions or the right sides of those limitations. The tasks with randomly variable coefficients of criteria functions are the most frequent and relatively simpler;
- **the nature of the solution of a model of a portfolio development** where the subject matter may be **optimization** or a certain kind of **reduction** of the initial set of the objects from which a portfolio is being created. As far as optimization is concerned, these tasks may be solved analytically by means of stochastic programming [29] supported by heuristic or genetic optimization algorithms based on the Monte Carlo simulation [24], [14] or by transformation to the deterministic equivalents of the tasks of stochastic programming. Of specific nature are the tasks based on the model (rule) of **the mean value – scatter** enabling to achieve **effective portfolios** maximizing the mean value of the yield criterion when the criterion is variably limited to the risk measured by scattering or standard deviation of this criterion (or if you like, minimizing the risk of portfolio when this criterion is variably limited to its yield). This approach is again based on the works of Markowitz (for more detail see, for example, [34]). **The reduction of a portfolio** leading to the determination of an effective set may be, again, based on the model (rule) of the mean value – scatter, or on the rules of the stochastic domination [10].

In this contribution the authors try to concentrate on the task of the optimization of a project portfolio under risk, partly by the solution of the deterministic equivalents of this task in the form of multi-criteria or mono-criteria bivalent programming under uncertainty in the assessment criteria, and partly by applying the optimization tool OptQuest for the stochastic optimization with continuous or discrete variables, based on linking the heuristic optimization algorithm with the Monte Carlo simulation [4], [25].

2. The Character of the Task of Creating a Project Portfolio

The core of the task of creating a project portfolio is creation of such a set of projects, out of a set of prepared projects, which meets certain conditions. The implementation of each project requires applying certain sources and that is why the task of creating a portfolio is also the task of **allocation of** (usually scarce) **sources**. If, in the process of creating a portfolio, our aim is maximization or minimization of certain characteristics of a portfolio, the task can be described as optimization in the form of an optimum allocation of sources.

The creation of a project portfolio has usually certain common features, namely the following ones:

- **The multi-criteria character** of the task, because more objectives are to be met and the level of accomplishment is expressed by means of the individual assessing criteria.
- Uncertainty of some factors influencing the results of the projects and therefore its success and hence the **risky nature** of the projects.
- **Scarcity of sources** meaning the fact that individual projects should not be considered in isolation as the acceptance of a certain project decreases disposable sources meant for other projects. This scarcity of sources calls for the need of using optimization tools.
- **Mutual dependency** of projects has to be taken into account too. The dependency of a portfolio can have the character of **statistical dependency** (direct or indirect dependency of various intensity expressed by means of correlation coefficients of the pairs of investment projects in the form of random quantities) or the character of **functional dependency** (e.g. a certain project can be put in a portfolio only if another particular project has also been put there).

3. The Optimization of a Project Portfolio under Risk by Means of the Deterministic Equivalents of the Task of Stochastic Programming

The deterministic equivalents of the tasks of the stochastic optimization transform these tasks to those of the deterministic type, when the divisions of the probability of the stochastic variables of the model are replaced by their statistical characteristics in the form of the mean values and scatterings.

In the following section (3.1) we will try and characterize the solution of the deterministic task of the multi-criteria optimization with randomly variable criteria by means of the multi-criteria utility function [18] as a tool of the multi-criteria assessment of the projects under risk, and also by means of the programme Lingo (for the solution of the tasks of linear and non-linear bivalent programming) [17]. In the next section (3.2) we will apply a similar approach by means of the mono-criteria optimization of a project portfolio under risk, and in the last section (3.3) we will try to characterize briefly other types of the deterministic equivalents of the tasks of the optimization of a portfolio under risk.

3.1 Multi-criteria Project Portfolio Optimization under Risk with Randomly Variable Criteria of Assessment

Task formulation

Let us assume that n projects have been prepared for the development of a company investment programme out of which it is necessary to draw up a portfolio maximizing its mean utility value from the point of view of m criteria when respecting p scarce sources. For each project there are divisions of probability of the individual assessment criteria available and their statistical characteristics in the form of the mean value and the standard deviation, the source demandingness and the disposable volumes of each source. (We can use the Monte Carlo simulation to determine the probability divisions of the quantitative criteria – for more detail see e.g. [8], [14] or [24], [25].)

Now, the deterministic equivalents of the task of the project portfolio optimization can be formulated as a task of **bivalent programming** with the criteria formulation based on the concept of multi-criteria utility function under risk and a set of source limitations. The construction of the multi-criteria utility function under risk is not easy and it requires the following:

- to verify preferential and utility independence of the criteria (only in this case it is possible to express the multi-criteria utility function under risk in the additive or multiplicative form);
- to construct partial utility functions $u_i(x_i)$ of the individual criteria and determine its weight in the dialogue of an analyst with a decision maker (for more detail see e.g. [6] and [18]),where the variable x_i expresses the consequence of the given project with regard to the *i*-th assessment criterion;
- to specify the form of the multi-criteria utility function under risk.

In case the requirement for the preferential and utility independence is met and the sum of the weights of the assessment criteria equals one (or at least approximately equals one), it is possible to express a multi-criteria utility function under risk in an additive form by the relation (1). In the opposite case it is necessary to express the multi-criteria utility functions under risk in a more complicated multiplicative form (for more detail see again [6] and [18]).

$$U(X) = \sum_{i=1}^{m} v_i . u_i(x_i). \qquad (1)$$

In case of the existence of more projects the utility of a *j*-th project $U(X^j)$ can be expressed by means of the relation (2):

$$U(X^j) = \sum_{i=1}^{n} v_i . u_i\left(x_i^j\right), \qquad (2)$$

where

X^j – vector of the consequence of *j*-th project with regard to the individual assessment criteria,

x_i^j – consequence of *j*-th project with regard to *i*-th assessment criterion (i =1, 2,…,m and j =1, 2,…,n),

$u_i(x_i^j)$ – utility of *i*-th consequence of *j*-th project,

v_i – weight of *i*-th criterion.

From the knowledge of the partial utility functions of the individual criteria and their division of probability we can derive the mean utility values E[$u_i(x_i^j)$] of each project with regard to i-th criterion. For a **continuous criterion** we use the relation (3):

$$E\left[u_i\left(x_i^j\right)\right] = \int_{d_i}^{h_i} u_i\left(x_i^j\right) . dF_i^j\left(x_i^j\right), \quad (3)$$

where

$F_i^j(x_i^j)$ – distribution fiction of j-th project with regard to i-th criterion,

d_i – lower limit of the definition domain of a partial utility function of the i-th criterion,

h_i – upper limit of the definition domain of a partial utility function of the i-th criterion.

In case of the assessment criteria of **discrete nature** the mean values of partial utilities can be set more easily as a sum of partial utilities weighed by their probabilities.

The knowledge of the mean values of partial utilities enables (in case of the additive form of the multi-criteria utility function) to set the **mean value of the overall utility** of the j-th project $E[U(X^j)]$ in the form (4):

$$E[U(X^j)] = \sum_{i=1}^{m} v_j . E[u_i(x_i^j)] \quad (4)$$

The mean value of the overall utility of the project portfolio can be now expressed easily as a sum of the mean values of the overall utilities of the individual projects contained in the portfolio. Let us define the bivalent variables:

$y_j \in \{0; 1\}$ gains the value 1 (j-th project is put to the portfolio) or 0 (j-th project is not put to the portfolio).

The task of the optimization of a project portfolio under risk can then be formulated as a **task of bivalent programming** with a criteria function expressing the mean value of the overall utility of the portfolio in the form (5):

$$\sum_{j=1}^{n} E[U(X^j)] . y_j \quad (5)$$

and with a set of source limitations as formulated in (6):

$$\sum_{j=1}^{n} a_k^j . y_j \leq L_k, \qquad k = 1, 2, \dots, p \quad (6)$$

where

a_k^j – consumption of k-th source for j-th project (k = 1, 2,…, p),

L_k – disposable volume of k-th source.

The model of bivalent programming can be extended further by limitations requiring the achievement of at least certain values of some quantities (let us indicate the set of the indexes of the quantities as Q) representing further requirements that the portfolio should meet (e.g. the requirement for achieving a certain size of sales, not exceeding the given risk and such like). The respective limitations for additive quantities of the yield type, among which there may also be some assessment criteria, might have the following form:

$$\sum_{j=1}^{n} h_i^j . y_j \geq V_q^{(min)}, \quad q \in Q \quad (7)$$

where

$V_q^{(min)}$ – the lower limit of q-th quantity of the yield type of the project portfolio,

h_i^j – the value of i-th quantity of the yield type of the j-th project (these are well known constants).

In case of the opposite requirement, i.e. non exceeding of the given values of certain quantities the relevant limitations should have an analogical form as the source limitations of the model of the bivalent programming.

An Example of the Solution of a Multi-criteria Project Portfolio Optimization under Risk

Let us demonstrate the solution of a task of multi-criteria project portfolio optimization under risk on an example of the project focused on introducing new products. Let us assume that for the development of this portfolio there are 12 projects available which were assessed with regard to the set of five criteria formed by two quantitative criteria (net present value (NPV) and cost effectiveness of the capital) and three qualitative criteria (concordance with the company strategy, market attractiveness and support of key competencies). The higher weight among the economic criteria went to the NPV; weights of the noneconomic criteria were judged. The division of the probability of the quantitative criteria for the individual projects was set by the Monte Carlo simulation; the division of probability of the qualitative criteria (transformed to discrete quantitative criteria) was done in an expert way. For each of the assessment criteria a partial utility function was specified. On the basis of these functions and the relevant divisions of probability as well as according to the relation (3), with continuous criteria or with analogical summative relation of discrete criteria, mean utility values were set for each project with regard to the individual criteria. By means of the relation (4) the mean values of the overall utilities were set.

The optimization of the project portfolio had also to respect two limited sources, namely the capital budget (CZK 560 million) and a disposable number of workers (240). The demands of the individual projects as far as the workers were concerned were set with a significant level of reliability and that is why it was possible to work with them as with deterministic quantities. The estimates of investment costs of the individual projects were considerably uncertain and that is why they were represented by random quantities with expertly set subjective divisions of probability. With regard to the fact that NPV represents a key financial criterion of the project assessment, even the calculation of the portfolio NPV in the form of its mean value was part of the optimization project (a sum of quantities for the project portfolio optimization is summarized in Table 1).

Tab. 1: Characteristics of Investment Projects

Project	E(U)	E(NPV) (mil CZK)	E(IN) (mil CZK)	Number of workers	$\frac{E(U)}{E(IN_{100})}$
Project 1	0.75	22.30	31.0	23	2.42
Project 2	0.36	5.16	27.6	12	1.30
Project 3	0.86	28.21	126.8	45	0.68
Project 4	0.59	26.32	96.5	37	0.61
Project 5	0.75	15.32	55.8	25	1.34
Project 6	0.78	10.47	36.8	23	2.12
Project 7	0.45	12.30	44.7	15	1.01
Project 8	0.65	23.01	67.5	26	0.96
Project 9	0.76	15.07	49.0	24	1.55
Project 10	0.48	29.47	85.8	38	0.56
Project 11	0.52	20.24	53.3	20	0.98
Project 12	0.74	12.35	42.3	17	1.75

Note: *E(U)* – mean value of the project overall utility,
E(NPV) – mean NPV value for a given project,
E(IN) – mean value of the investment costs of a given project,
$\frac{E(U)}{E(IN_{100})}$ – mean value of a project utility related to 100 million of investment costs spent (IN) expressed again by their mean value.

Source: own calculations

If all the prepared projects were put in the portfolio, the limited sources would be exceeded considerably. (The demandingness of the implementation of the portfolio on the funds would amount CZK 717.1 million, which means exceeding the limit of CZK 560 million by CZK 157.1 million and the disposable number of workers of 240 would be exceeded by 65; the mean utility value of this portfolio would be 7.69 and the mean NPV value would be CZK 220.2 million.) Therefore a deterministic equivalent of optimization of a project portfolio under risk was used for the development of the portfolio, which has a form of a bivalent programming model. The criteria function of this model maximizing the mean utility value of the portfolio is given by the relation (5) and its two source limitations relating to the capital budget and the disposable number of workers are given by the relation (6). By solving the optimization task by means of the software tool for bivalent linear and non-linear programming LINGO (see for example [17]) we gained an optimum portfolio created by projects 1, 3 up to 6, 8, 9, 11 and 12. The mean utility value of this portfolio was 6.40, the mean NPV value was CZK 173.3 million and almost all the budget was drawn (only CZK 1 million remained disposable) and all 240 workers were fully used (see the solution in Table 2 for the portfolio A).

Another subject of interest consisted in finding out what impacts an increase of the requirement concerning the mean NPV value would have on the optimum portfolio. Two tasks of bivalent programming were gradually solved with the requirement to achieve the mean NPV value of at least CZK 175, or, as the case may be, CZK 180 million. The results of both these optimizations (portfolio B and portfolio C) together with the results of optimization without the limitation to the mean value NPV (portfolio A) are summarized in Table 2.

Tab. 2: The Results of the Portfolio Optimization

Portfolio	Portfolio structure	E (U)	E (NPV)	Utility decrease		NPV growth		Source demands	
				abs.	%	mil CZK	%	F	W
A	1, 3–6, 8, 9, 11, 12	6.40	173.3					559	240
B	1, 2, 4, 6–12	6.08	176.7	0.32	5	3.4	2.0	535	235
C	1, 2, 4, 5, 7–12	6.05	181.5	0.03	0.5	4.8	2.7	554	237

Note: *E(U)* – mean utility value of the project portfolio,
E(NPV) – mean NPV value of the project portfolio,
F – funds,
W – number of workers.

Source: own calculations

As is obvious from this table the requirement for the mean value to achieve at least CZK 175 million (portfolio B) leads to the decrease of the mean utility value by 0.32 (5 %), while the growth of the mean NPV value is only CZK 3.4 million (2 %). Further tightening of the requirement for NPV (achieving at least CZK 180 million, portfolio C) leads to a considerable smaller decrease of the mean utility value (absolutely 0.03 or 0.5 %) as opposed to portfolio B, while the growth of the mean value NPV is now higher (CZK 4.8 million or 2.7 %).

The results of the optimization calculations can now serve as a valuable source of information of a company management for the decision about the structure of the portfolio to be implemented. In this decision making process it is also necessary to consider other aspects not included in the optimization, including the change in drawing the limited sources (e.g. the idle CZK 25 million with portfolio B).

The optimization of the project portfolio could be, to a certain extent, simplified in our task in case of the existence of only one limited source. If this was the capital budget, we could use the indicator $E(U) / E(IN_{100})$ whose values for all projects are stated in Table 1. If we ordered the projects according to the decreasing values of this indicator and gradually put the projects according to the decreasing values of this indicator down to achieving the capital limit, we would, in this simple way, proceed to a relatively good portfolio. In case of the requirement for finding an optimum portfolio this task may lead to the solution of the task called a knapsack problem, which is a relatively simple task of integer linear programming. If we, in our case, applied the above simple procedure, the projects 4 and 10, i.e. those with the lowest mean utility values at CZK 100 million of investment costs, would not get any funding. This way of optimization of the research of the new medicaments has been applied by one of the world's biggest pharmaceutical companies GlaxoSmithkline. The mean NPV value on USD 1 million serves as the optimization criterion (for more detail see [30]).

What is seen as a considerable disadvantage of the multi-criteria project portfolio optimization under risk is the difficulty of constructing partial utility functions, which managers do not like to work with. Applying the *Cost Benefit*

Analysis (CBA) – (see [5]) may be a solution as in some cases this method enables a transfer of criteria values expressed in the non-monetary units to a monetary expression and this way the monetary assessment can be integrated for example to the indicator of the net present value (NPV) which might serve as an overall criterion of the project portfolio optimization. Another possibility is the transfer of certain criteria of assessment to the limitation conditions, i.e. especially those criteria which were not possible to transform to a monetary expression. This procedure is not complicated as it only leads to increasing the number of limitations of the optimization model for which it is necessary to set their right sides in the form of the lower limits (for the criteria of the yield type) or the upper limits (for the criteria of the cost type). In practice and sometimes even in professional literature (for example [22]), we can come across attitudes where even the project risk is included in the assessment criteria. The main drawback of this attitude based on the multi-criteria project assessment as if carried out in the conditions of certainty (i.e. on the basis of only one scenario) is that it does not respect the dependency of the project assessment with regard to some criteria under this type of risk (a higher risk only decreases the overall assessment of projects).

3.2 Mono-criteria Project Portfolio Optimization under Risk with a Randomly Variable Assessment Criterion

The model of bivalent programming for the mono-criteria optimization of a project portfolio under risk can be easily derived from the model of the multi-criteria assessment. Let us assume that a key additive optimization criterion c of the yield type has been chosen. Further, we assume that the mean values $E(c_j)$ for the individual projects ($j = 1, 2, \dots, n$) are known and the bivalent variables are defined as $y_j \in \{0; 1\}$, where for $y_j = 1$ j-th project is put to portfolio and for $y_j = 0$ is not included in the portfolio. Then the criterion function of the deterministic equivalent of the stochastic optimization of development of a project portfolio under risk can be expressed as (8):

$$\sum_{j=1}^{n} E(c_j) \cdot y_j \rightarrow max \tag{8}$$

The set of the source limitations of the model or the limitations expressing the requirements for non-exceeding of certain values of the chosen quantities of the cost type model can be expressed by means of the relation (6). The same way the requirements for achieving at least certain values of the chosen quantities of the yield type model can be expressed by means of the relation (7).

In the tasks concerning the project portfolio optimization under risk, we have not yet, within the set of the model limiting conditions, included the limitation concerning the risk of a project portfolio expressed in relation to the optimization criterion. The optimal portfolio maximizing (in case of the yield type criterion) the mean value of this portfolio might be considerably risky as the more risky projects usually lead to higher yields. With regard to this it is necessary to extend the set of limitations of the bivalent programming model by the limitation of the portfolio risk in the form of (9):

$$\sigma = \left(\sum_{i=1}^{n} y_j \cdot \sigma_i^2 + \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} y_i \cdot y_j \cdot r_{ij} \cdot \sigma_i \cdot \sigma_j\right)^{1/2} \leq \sigma_h, \tag{9}$$

where

- σ – standard deviation of the optimization criterion of the project portfolio,
- σ_i, σ_j – standard deviation of the optimization criterion of the i-th or j-th project,
- σ_h – upper, limit of the portfolio risk with regard to the set-down optimization criterion as expressed by the standard deviation,
- r_{ij} – correlation coefficient of the value of i-th or j-th project (these values can be estimated expertly on the basis of the proximity or dissimilarity of the subject matter of the projects, their market determination, raw material base and such like, while a role is also played by the relations of the systematic and specific risks of the individual projects).

It is obvious that the less strict the limitations (9) of risks are, the higher the mean value of the portfolio is. It might now be useful to find out about the impacts of the tightening up of the requirements for the portfolio risk as for the structure and characteristics of the set-down optimal (effective) portfolios. The graphic depiction of these portfolios then forms the **efficient frontier** (see below).

3.3 Further Types of Deterministic Equivalents of the Tasks of a Project Portfolio Optimization under Risk

In this section we are going to outline further types of the tasks of the optimization of a project portfolio under risk, partly by maximizing the probability of exceeding the target value of the criterion, and partly by a task with randomly variable limitations.

Maximization of the Probability of Exceeding the Target Value of a Criterion

Let us assume that NPV is the criterion of the project portfolio assessment and its target (planned) value NPV_C has been set. On condition that the variables y_j are identical with those in the text above, the criteria function of the project portfolio under risk can now be written down in the form of (10):

$$P\left(\sum_{j=1}^{n} y_j \cdot NPV_j \geq NPV_c\right) \rightarrow \max \quad (10)$$

where

NPV_j – the net present value of j-th project (j =1,2,...,n),

NPV_C– target value of the net present value of the project portfolio.

The limiting conditions expressing the source settings are possible to express by the relation (6) and the limitations relating to the risk of the portfolio by the relation (9). In case that NPV_j can be expressed as a linear function of a random parameter $\delta \in \langle 0; 1 \rangle$ in the form of (11):

$$NPV_j = NPV_j^{(0)} + \delta \cdot NPV_j^{(1)} , \quad (11)$$

the deterministic equivalent of the task of maximization of the probability of exceeding the target value NPV_C of the portfolio can then be transformed to a task of linear fractional programming with a criteria function in the form (12):

$$\frac{NPV_C - \sum_{j=1}^{n} NPV_j^{(0)} \cdot y_j}{\sum_{j=1}^{n} NPV_j^{(1)} \cdot y_j} \rightarrow \min \quad (12)$$

with limitations (6) and (11). A solution of this task using simulation is given in section 3.3.

Optimization of the Project Portfolio under Risk with Randomly Variable Limitations

The following factors can be the randomly variables in this case:

- the right sides of the limiting conditions (usually of source limitations), and
- the coefficients of the variables on the left sides of the limitations (in case of source limitations it is the demandingness of the individual projects on the limited sources).

The tasks with random variables on the right sides of the limitations are easier to solve. In case of the source limitations it may be an example of export activities from a remote foreign country, raw material in the initial stage of extraction where a geological survey of a deposit thickness has not been completed yet and such like.

If, for example, the disposable volume of k-th limited source is uncertain and we know its division of probability, then a part of the related optimization model will be for example a limitation requiring that the probability that the consumption of k-th source will not exceed its mean value will be higher than the set value. An appropriate limitation should then have the form of (13):

$$P\left[a_k^j \leq E(L_k)\right] \geq \alpha_k, \quad (13)$$

where

a_k^j – consumption of k-th source for j-th project,

$E(L_k)$ – mean value of the disposable volume of k-th source,

α_k – lower limit of the probability of non-exceeding the mean value of the disposable volume of k-th source in percentage.

For the deterministic equivalents of the task of optimization of the project portfolio it is now possible to transform the above stated limitation (see [12]) to the form (14):

$$\sum_{j=1}^{n} a_k^j \cdot y_j \leq L_k^*, \quad (14)$$

where

L_k^* – the (1 – α_k) percentile of the division of the probability of disposable volume of the k-th limited source.

In the tasks of the optimization of a project portfolio under risk we can come across the uncertainty of the coefficients α_{kj} of certain limitations of the model more often. The limitation of the capital budget of a portfolio can be a typical example of this, when the investment costs of the individual projects are considerably uncertain. The experience from business practice shows that estimates of these costs are considerably optimistic and they are usually exceeded – see [9]. If both these coefficients and the right sides of the relevant limitations (volumes of disposable incomes) are random, then it is possible to transform them to the set of non-linear limitations, and so the deterministic equivalent of the optimization of a project portfolio under risk has the form of a model of non-linear bivalent programming [12].

4. The Application of the Monte Carlo Simulation in the Process of the Stochastic Optimization of a Project Portfolio

As we have already stated, certain simpler tasks of the optimization of the project portfolio under risk are possible to solve by their transfer to deterministic equivalents. Difficulties arise in case of more difficult tasks (not only the coefficients of criteria function are of a random nature but also the coefficients of the left sides of the limiting conditions, and their right sides), or, as the case may be, in other types of the criteria functions than the functions expressed by relations (5) and (8). Analytic solution of these optimization tasks of stochastic bivalent programming is usually considerably difficult, but the above mentioned optimization programs can be applied successfully.

Gradually four tasks of stochastic optimization of the project portfolio were solved based on an example whose deterministic equivalent was solved in section 3.1. In this model there are, on the whole, 24 random variables which represent NPV and the investment costs for each model. The division of the probability of NPV of the individual projects were set by the Monte Carlo simulation, and these divisions were approximated by the most suitable types of the theoretical divisions (with five projects it was normal divisions, with the remaining seven projects it was beta division with negative skewness. The division of the investment costs probability has mostly the character of betaPERT division (with expertly estimated parameters by authors of this article) and positive skewness. As a basis of determination of the probability division of these costs it is possible to use the post-audit results of similar projects that are focused on detection and evaluation of deviations between planned and for real expended investment costs. For more detail see [9] and [31].

With regard to the fact that the investment costs are one of the significant factors influencing NPV of each project, statistical dependency of NPV and the costs were respected for each project (the expertly estimated correlation coefficients on the basis of the investment costs contributions to the uncertainty of NPV were set by the disperse analysis and they were from -0.2 to -0.4). The model of this task is given by Table 3. First we paid attention to the optimization of the portfolio while the limitation of the risk was variable and the risk was expressed by a standard deviation NPV leading to **effective portfolios**, or, as the case may be, to an **efficient frontier** (section 4.1). The task of setting possible impacts of **increasing** or **decreasing** disposable volume of certain limited sources on the structure and the effects of the portfolio dealt with in section 4.2 is conceptually similar and also significant from the practical point of view. Further two tasks aim partly at the **maximization** of exceeding the target value of the optimization criterion, in our case it is NPV (section 4.3), and partly at the **minimization** of exceeding the capital budget as a key limitation of the development of a project portfolio under risk (section 4.4).

4.1 Efficient Frontier

By means of the optimization tool OptQuest five tasks of stochastic optimization were solved, with a criteria function formed by the mean NPV value, two source limitations (the capital budget CZK 560 million and the number of workers not exceeding 240) and the gradually released portfolio risk limitations with regard to NPV measured by its standard deviation. The upper limit of the standard deviation was gradually increased by CZK 1 million, from CZK 5 million to CZK 9 million. During the time of the task solving in the length of two minutes the programme came up with 2,304 solutions, out of which 1,816 were feasible. The results of the solution are summarized in Table 4.

Tab. 3: A Model of an Investment Portfolio

Project	E(NPV) (mil CZK)	σ(NPV) (mil CZK)	E(IN) (mil CZK)	σ(IN) (mil CZK)	Number of workers
Project 1	22.30	3.20	31.0	2.30	23
Project 2	5.16	1.62	27.6	1.70	12
Project 3	28.21	6.67	126.8	4.10	45
Project 4	26.32	4.35	96.5	6.80	37
Project 5	15.32	1.53	55.8	4.60	25
Project 6	10.47	1.80	36.8	3.00	23
Project 7	12.30	0.94	44.7	3.80	15
Project 8	23.01	2.75	67.5	4.30	26
Project 9	15.07	2.11	49.0	3.30	24
Project 10	29.47	5.32	85.8	5.50	38
Project 11	20.24	3.91	53.3	4.70	20
Project 12	12.35	3.12	42.3	3.10	17

Note: *E(NPV)* – mean value NPV for a given project,
σ *(NPV)* – standard deviation NPV for a given project,
E(IN) – mean value of investment costs of a given project,
σ *(IN)* – standard deviation of investment costs of a given project.

Source: own calculations

Tab. 4: The Results of Stochastic Optimization with Variable Limitations of a Portfolio Risk

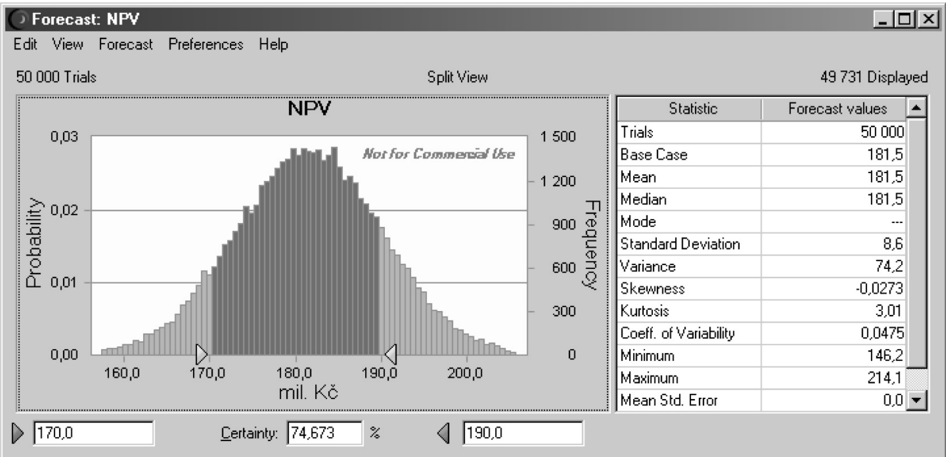
Effective portfolios	Portfolio structure	E(NPV)	σ(NPV)	E(U)	E(IN)	P	$\frac{\Delta E(NPV)}{\sigma\Delta(NPV)}$
EP1	4–7, 9	99.7	4.9	3.85	336	144	
EP2	4–10	132.0	5.8	4.46	436	188	35.9
EP3	1, 2, 4–10	159.5	6.8	5.57	495	223	27.5
EP4	1,2, 4–10, 12	171.8	7.5	6.31	537	240	17.6
EP5	1,2, 4, 5, 7–12	181.5	8.6	6.05	554	237	8.9

Notes: see Tables 1 and 3

Source: own calculations

As is obvious from Table 4, with gradual softening of the requirements concerning the portfolio risk measured by the standard deviation σ(NPV), the mean value NPV of portfolios increases and their mean utility value also grows. With the first two effective portfolios their risk limitation does not enable the full exploitation of sources. The last two effective portfolios lead to almost full exploitation of sources. The last effective portfolio EP5 leads to the maximum mean NPV value. The effective portfolio EP4 reaches a higher mean utility value with a lower risk. When choosing between these two effective portfolios it should be, therefore, necessary to consider the growth of the mean NPV value when transferring from portfolio EP4 to portfolio EP5 on one hand, and the risk increase and the decrease of the mean utility value on the other hand. The division of the probability of the effective portfolio EP5 with the maximum mean value of its NPV is illustrated by Figure 1.

Fig. 1: Division of NPV Probability of Portfolio 5

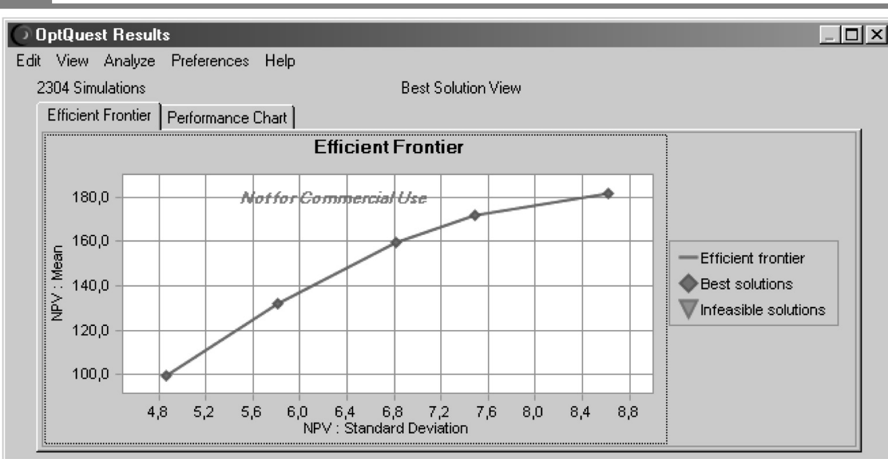


Source: own calculations

As is obvious from Figure 1, NPV of this portfolio varies between CZK 146.2 million to CZK 214.1 million, and the probability that the NPV will reach the value from the interval from CZK 170 million to CZK 190 million is approximately 75%. The skewness -0.0273 demonstrates a slight deviation of this division towards the lower NPV values. The balancing of this division by the most suitable of the theoretical division according to Anderson-Darling test led to a normal division with identical parameters of its division as set by the simulation.

From Table 4 it is also obvious that by increasing the risk of the effective portfolios the growth of the NPV mean value on a risk growth unit as measured by a standard NPV deviation decreases (see column *ΔE (NPV)/Δσ(NPV)*. This fact is also confirmed by the graphic image of the effective portfolios in the form of the efficient frontier on Figure 2, when the slopes of the line segments connecting the individual effective portfolios decline from the left to the right. See [32], [33] for more about this approach.

Fig. 2: Efficient Frontier



Source: own calculations

4.2 The Influence of the Source Limit Increase on the Portfolio Effects

As is obvious from Table 4, the effective portfolio EP5 with the maximum mean NPV value leads to almost a full use of the limited sources. It might, therefore, be useful to find out to what extent gaining additional sources might increase the effects of the portfolio optimization.

The chosen results of stochastic optimizations with a somewhat increased capital budget and the limit of workers (again, the mean NPV value was the optimization criterion) are stated in the Table 5. The portfolio D represents the optimum (efficient) portfolio with the given capital budget (CZK 560 million) and with the limit of the number of workers (240), and it is identical with the efficient portfolio EP5 from the Table 4. Even a slight increase of the capital budget (by CZK 3 million) and of the number of workers (by 8) leads to the optimum portfolio E with a higher NPV and utility mean values. Another optimum portfolio F shows a higher increase of the capital budget (by CZK 30 million) and the limit of workers (by 20).

Tab. 5: The Influence of the Increase of Source Volume on the Optimum Portfolios

Port-folios	E (IN)	P	Portfolio structure	E (NPV)	σ (NPV)	$\frac{E(NPV)}{E(IN)}$	$\frac{E(NPV)}{P}$	E (U)	$\frac{E(U)}{E(IN_{100})}$	$\frac{E(U)}{P_{100}}$
D	554	237	1, 2, 4, 5, 7–12	181.6	8.6	0.327	0.77	6.05	1.09	2.55
E	563	248	1, 4–12	186.9	8.7	0.331	0.75	6.47	1.15	2.61
F	590	260	1, 2, 4–12	192.0	8.8	0.325	0.74	6.83	1.16	2.63
G	608	250	1, 3–5, 8–12	192.3	9.6	0.315	0.74	6.10	1.00	2.44

Notes: see Table 1 and Table 3 Source: own calculations

In case of difficulties in increasing the limit of the restricted sources it would probably be possible to implement the portfolio E that, with regard to the portfolio D, leads to the increase of the mean NPV value and the mean utility value to CZK 100 million of investment costs and to only to a slight increase of risk and a slight decrease of the mean NPV value on one worker. A higher availability of limited sources (portfolio F) leads only to a very small increase in the mean utility value on the spent CZK 100 million and on one hundred workers, by decrease of similar indicators related to NPV but it also leads to an insignificant risk increase. The last portfolio G requires, on top of portfolio F, gaining CZK 18 million and at the same time reducing the necessary number of workers by 10. Most indicators characterizing this portfolio in Table 5 shows deterioration. Depending on the possibilities of the increase of the volume of limited sources portfolios E or F are possibly the best to be taken into account.

4.3 Maximization of the Probability of Exceeding the Criterion Target Value

Three optimization tasks were gradually solved with target NPV values amounting CZK 175 million, CZK 180 million and CZK 185 million and with two source limitations (the capital budget and the number of workers). The results of the solution are shown in Table 6. As is obvious from this table, the optimum portfolios H, I and J maximizing the probability of exceeding the target NPV values are identical and are formed by ten out of twelve projects, while projects 3 and 6 were excluded.

The decrease of the probability of exceeding the target value (from 77.4 % with portfolio H to 34.2 % with portfolio J) is the obvious consequence of increasing the target NPV value of the portfolio.

The first three tasks were solved without any limitations related to the mean utility value of the portfolio. If we required this value to

Tab. 6: The Results of Maximization of the Probability of Exceeding the Target NPV Value

Portfolio	Target value NPV (NPV_c)	Portfolio structure	$P(NPV \geq NPV_c)$	E(NPV)	σ(NPV)	E(U)
H	175	1, 2, 4, 5, 7–12	77.4	181.5	8.6	6.05
I	180	1, 2, 4, 5, 7–12	57.0	181.5	8.6	6.05
J	185	1 , 2, 4, 5, 7–12	34.2	181.5	8.6	6.05
K	180	1, 2, 4–10, 12	13.2	171.8	7.5	6.31

Source: own calculations

reach at least 6.2, then with the target NPV value amounting CZK 180 million there is a change of the optimum portfolio, portfolio K in this case. The mean utility value of this portfolio reaches the value 6.31, but there was a considerable decrease of the mean NPV value of the portfolio by approx. CZK 10 million. The probability of exceeding the target NPV value by this portfolio is considerably low (only 13.2%) and so it is highly probable (86.8 %) that this value is not going to be reached.

4.4 Minimization of the Probability of Exceeding the Capital Budget

The individual portfolios may vary by the probability of exceeding the capital budget and therefore this criterion can represent one of the criteria of the portfolio assessment. In our case we therefore solved a few optimization tasks of the minimization of the probability of exceeding the capital budget amounting CZK 560 million with a gradually growing requirement for the mean NPV value of the portfolio. The results of these tasks solution are summarized in Table 7 (only mutually different solutions are stated here).

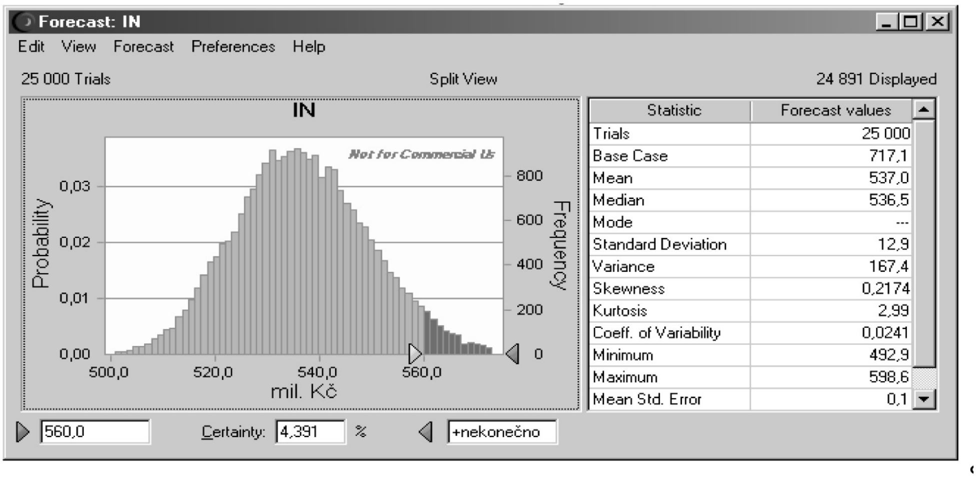
Tab. 7: The Results of Minimization of the Probability of Exceeding the Capital Budget

Portfolio	Lower limit E(NPV)	P (IN ≥560) (%)	E(IN)	E(NPV)	σ(NPV)	E(U)	Projects not included in portfolio
L	171	0.01	507	171.5	9.0	5.72	2, 3, 5
M	171	4.39	537	171.8	7.5	6.31	3, 11
N	175	0.80	526	176.4	8.5	5.69	2, 3, 6
P	177	25.94	532	177.0	9.2	5.35	2, 5, 6, 7
R	180	30.48	554	181.5	8.6	6.05	3, 6

Source: own calculations

The portfolio L leads to an absolutely negligible exceeding of the investment costs but it has a considerably high risk as measured by a standard NPV deviation. If we required this risk not to increase CZK 8 million, we would gain portfolio M with approximately the same NPV value but with a considerably higher mean utility value (6.31). The probability that by implementing this portfolio the capital budget will be exceeded is less than 5%. Out of other solutions there is portfolio N that leads to a very small exceeding of the capital budget (0.80 %), its mean NPV value being approx. by CZK 5 million higher than with portfolio M but there was a considerable decrease in the mean utility value (from value 6.31 to 5.69). With other portfolios, P and R, and considering higher requirements for the mean NPV value, there is a considerable increase of the probability of exceeding the capital budget. Figure 3 shows the investment costs division probability of the low risk portfolio M.

Fig. 3: **The Investment Costs Division Probability of Portfolio M**



Source: own calculations

4.5 Summary of the Results of the Optimization of a Project Portfolio under Risk

The results of the solution of the individual optimization tasks now enable to select suitable solutions and to assess them with regard to the selected set of criteria. If we somewhat simplify the portfolio assessment and consider the capital budget as the only limiting source, then, with regard to a certain difference in the demandingness of the individual portfolios on the funds, it might be more appropriate not to work with the absolute criteria in the form of the mean utility value or the NPV mean value but with the relative values in the form of the mean utility or NPV value as related to the mean value of the unit of the investment costs spent. As for other suitable criteria for the portfolio assessment we might also consider the risk with regard to the NPV expressed by the standard deviation, the probability of exceeding the NPV target value and the probability of exceeding the disposable volume of funds.

The following selected results of the optimization tasks were the assessed portfolios whose common thread was the portfolio structure.

- Port1 corresponds to portfolios C, D, I, R and EP5 and it does not contain projects 3 and 6.
- Port2 corresponds to portfolios K, M, EP4 and it does not contain projects 3 and 11.
- Port3 corresponds to portfolio E and it does not contain projects 2 and 3 and its specific nature consists in the fact that the capital budget increases by CZK 3 million, to CZK 563 million, and the number of workers increases by 8, to 248.

The values of the portfolio criteria together with the mean values of utility and NPV are shown in Table 8.

Tab. 8: **The Criteria Values of Selected Portfolios**

Portfolio	$\frac{E(U)}{E(IN_{100})}$	$\frac{E(NPV)}{E(IN)}$	σ (NPV)	P(NPV≥180) (%)	P(IN≥560) (%)	E(U)	E(NPV)	E(IN)
Port1	1.09	0.327	8.6	57.0	30.5	6.05	181.5	554
Port2	1.18	0.320	7.5	13.2	4.4	6.31	171.8	537
Port3	1.15	0.331	8.3	78.4	47.3	6.48	186.9	563

Source: own calculations

It may now be possible to carry out the selection of the most suitable portfolio either in the **non-formalized way** or by applying the methods of the **multi-criteria assessment.**

- In case of the non-formalized approach we would first compare the individual portfolios from the individual criteria point of view, e.g. Port3 is better than Port1 from the point of view of the mean utility and NPV value, but they are approximately at the same level of risk etc. The choice of the portfolio, as far as the risk is concerned, would be influenced by the attitude of the decision maker.
- The formalized assessment of portfolios would require setting the significance of the individual criteria in the form of their weights and applying some methods of the multi-criteria assessment (e.g. [10]). This approach is suitable especially in case of a greater number of the assessed portfolios and a larger set of assessment criteria, where the non-formalized approach may fail. In our task it would also be necessary to respect the strong interconnection of the two following criteria, namely the size of the mean NPV value of portfolios and the probability of exceeding the NPV target value.

Conclusion

The development of the project portfolios in the form of an investment programme or a research programme in the business practice is accompanied by a lot of drawbacks. Here are the most significant ones:

- the way of handling risk (the portfolio is mostly created in the quasi conditions of certainty; i.e. on the basis of the only one possible scenario of the future development);
- the projects are assessed and included into the portfolio independently on one another; i.e. their mutual relations either of the deterministic or stochastic character are not respected;
- the multi-criteria character of the task is only seldom respected in the optimization of the portfolio.

These and some other drawbacks of the portfolio development, including for example the absence of the projects interlinked with the company strategic objectives, non-respecting the various levels of risk of the individual projects, the imbalance of the portfolio from the point of view of the represented project types, the intuitiveness of the creation without applying the analytic tools based on quantitative data are pointed out by for example [19], [21], [20], [27] and [26]. The political character of the process of the portfolio development can also have negative impacts on the company effectiveness. Instead of handling this issue as a rational process considering the source demandingness of the individual projects, their corporate benefits, risks and mutual dependencies it is rather a process that is insufficiently transparent, aimed at pushing local interests, compromising, demonstration of power (Sanwall [28] calls this process Decibel-Driven) and such like.

The above drawbacks can be eliminated, or at least reduced by the application of the optimization models, in which we characterized partly the optimization of a project portfolio development under risk in terms of the deterministic equivalents of this task by means of some models of bivalent programming, and partly the stochastic optimization based on the Monte Carlo simulation. Gaining suitable IT support may not be enough for practical application of these tools, which undoubtedly provide invaluable information for creating investment or research programmes, but it is also necessary to create the needed personnel and organisational prerequisites (including providing the necessary know-how) and to overcome the resistance towards change by means of suitable motivation and first of all with the help of the company top management (for further detail see [14]). With regard to the fact that the decision making process concerning the investment programmes or research programmes belongs to the key decisions of a strategic nature, the optimization of a project portfolio development under risk may become an important factor of the growth of effectiveness and prosperity in any company.

The article was worked out as one of the outputs of the research project called "Competitiveness" (registration number VSE IP300040) solved at the University of Economics, Prague, Faculty of Business Administration funded by institutional support of the long-term conceptual development at research institutions.

References

[1] ABDELAZIZ, F.B., AOUNI, B., EL FAYEDH, R.E. Multi-objective stochastic programming for portfolio selection. *European Journal of Operational Research*. 2007, Vol. 177, Iss. 3, pp. 1811–1823. ISSN 0377-2217.

[2] ALESSANDRI, T.M., FORD, D.N., LANDER, D.M., LEGGIO, K.B, TAYLOR, M. Managing risk and uncertainty in complex capital projects. *The Quarterly Review of Economics and Finance*. 2004, Vol. 44, Iss. 5, pp. 751–67. ISSN 1062-9769.

[3] BALLESTERO, E., GUENTHER, M., PLA-SANTAMARIA, D., STUMER C. Portfolio selection under strict uncertainty: A multi-criteria methodology and its application to the Frankfurt and Vienna Stock Exchanges. *European Journal of Operational Research*. 2007, Vol. 181, Iss. 3, pp. 1476–1487. ISSN 0377-2217.

[4] CHARNES, J. *Financial Modeling with Crystal Ball and Excel*. Hoboken: J. Wiley & Sons, 2007. ISBN 978-0-471-7792-8.

[5] European commission. *Guide to Cost-Benefit Analysis of Investment Projects.* Brussels: European Commission. Directorate General Regional policy, 2008.

[6] FOTR, J., PÍŠEK, M. *Exaktní metody ekonomického rozhodování.* 1. vyd. Praha: Academia, 1986. 165 s.

[7] FOTR, J., SOUČEK, I. *Investiční rozhodování a řízení projektů*. Praha: Grada Publishing, 2011. ISBN 978-80-247-3293-0.

[8] FOTR, J., ŠVECOVÁ, L., SOUČEK, I., PEŠÁK, L. Simulace Monte Carlo v analýze rizika investičních projektů. *Acta Oeconomica Pragensia.* 2007, Vol. 15, Iss. 2. pp. 32–43. ISSN 0572-3043.

[9] FOTR, J., ŠVECOVÁ, L., ŠPAČEK, M. Postaudity investičních projektů jako nástroj učení se. *Ekonomický časopis*. 2009, Vol. 57, Iss. 7, pp. 633–652. ISSN 0013-3035.

[10] FOTR, J., ŠVECOVÁ, L. et al. *Manažerské rozhodování. Postupy, metody a nástroje*. Praha: Ekopress, 2010. ISBN 978-80-86929-59-0.

[11] GOLOSNOY, V., OKHRIN, Y. General uncertainty in portfolio selection: A case-based decision approach. *Journal of Economic Behavior & Organization*. 2008, Vol. 67, Iss. 3–4, pp. 718–734. ISSN 0167-2681.

[12] GROS, I. *Kvantitativní metody v manažerském rozhodování.* Praha: GradaPublishing, 2003. ISBN 80-247-0421-8.

[13] GUPTA, P., MEHLAWAT, M.K., SAXENA, A. Asset portfolio optimization using fuzzy mathematical programming. *Information Sciences*. 2008, Vol. 178, Iss. 6, pp. 1734–1755. ISSN 0020-0255.

[14] HNILICA, J., FOTR, J. *Aplikovaná analýza rizika ve finančním managementu a investičním rozhodování*. Praha: Grada Publishing, 2009. ISBN 978-80-247-2560-4.

[15] HANAFIZADEH, P., KAZAZI, A., BOLHASANI, A.J. Portfolio design for investment companies through scenario planning. *Management Decision*. 2011, Vol. 49, Iss. 4, pp. 513–532. ISSN 0025-1747.

[16] HUANG, X. Two new models for portfolio selection with stochastic returns taking fuzzy information. *European Journal of Operational Research*. 2007, Vol. 180, Iss. 1, pp. 396–405. ISSN 0377-2217.

[17] JABLONSKÝ, J. *Operační výzkum. Kvantitativní modely pro ekonomické rozhodování*. 3. vyd. Praha: Professional Publishing, 2007. 323 s. ISBN 978-80-86946-44-3.

[18] KEENEY, R., RAIFFA, H. *Decisions with Multiple Objectives: Preferences and Value TradeOffs*. Cambridge: Cambridge University Press, 1993. ISBN 0-521-43883-7.

[19] KENDALL, G., ROLLINS, S. *Advanced Portfolio Management and the PMO*. Boca Raton: Ross Publishing, 2003. 434 p. ISBN 978-1932159028.

[20] KISLINGEROVÁ, E. et al. *Vyhodnocení dotazníků*. Interní materiál Fakulty podnikohospodářské VŠE v Praze zpracovaný v rámci řešení výzkumného záměru MSM 6138439905. Praha: VŠE, 2008.

[21] KODUKULA, P. *Project Portfolio Management. Practitionerįs Approach.* Podklady ze semináře. Praha: Stamford Global, 2007.

[22] KOLLER, G. *Risk Assessment and Decision Making in Business and Industry*. 2. vyd. Boca Raton: Chapman&Hall/CRC, 2005. 352 p. ISBN 978-1584884774.

[23] MARKOWITZ, H.M. Portfolio selection. *Journal of Finance*. 1952, Vol. 7, No. 1, pp. 77–91. ISSN 1540-6261.

[24] MUN, J. *Applied Risk Analysis*. New York: John Wiley & Sons, 2004. ISBN 978-0471478850.

[25] MUN, J. *Modelling Risk.* New York: John Wiley & Sons, 2006. ISBN 978-0471789000.

[26] LEVINE, H. *Project Portfolio Management. A Practical Guide to Selecting Projects, Managing*

Portfolios and Maximising Benefits. San Francisco: Jossey Bass, 2005. 560 p. ISBN 978-0787977542.
[27] PENNAPACKER, J., RETNA, S. *Project Portfolio Management. A View from Management Trenches*. Hoboken: John Wiley & Sons, 2009. 256 p. ISBN 978-0470505366.
[28] SANWAL, A. *Optimizing Corporate Portfolio Management. Aligning Investment Proposals with Organizational Strategy*. Hoboken: John Wiley & Sons, 2007. ISBN 978-0-470-12688-2.
[29] SHAPIRO, A., DENTCHEVA, D., RUDZINSKI, A*. Lectures on stochastic programming. Modelling and Theory.* Philadelphia: SIAM-Society for Industrial and Applied Mathematics, 2009. ISBN 978-0-8987316-87-0.
[30] SHARPE, P., KEELIN, T. How SmithKline Beecham Makes Better Resource Allocation Decisions. *Harvard Business Review*. 1998, Vol. 76, Iss. 2, pp. 45. ISSN 0017-8012.
[31] ŠPAČEK, M., SOUČEK, I. *Postaudity investičních projektů*. In KISLINGEROVÁ, E. a kol. *Nová ekonomika, nové příležitosti?* Praha: C. H. Beck, 2011. ISBN 978-80-7400-403-2.
[32] ŠVECOVÁ, L., FOTR, J. Project Portfolio Development Under Risk And Its Optimization. *Research Journal of Economics, Business and ICT*. 2011, Vol. 3, pp. 64–69. ISSN 2045-3345. Available also on: <www.researchjournals.co.uk/documents/Vol3/14.pdf>.
[33] ŠVECOVÁ, L., FOTR, J. The Optimization of a Project Portfolio Development Under Risk. In *International Days of Statistics and Economics*. Prague, 2011. September 22–23. pp. 615–624. Available also on: <msed.vse.cz/files/2011/Svecova.pdf>.
[34] ZHANG, W.G., WANG, Y.L., CHEN, Z.P., NIE, Z.K. Possibilistic mean-variance models and efficient frontiers for portfolio selection problem. *Information Sciences*. 2007, Vol. 177, Iss. 13, pp. 2787–2801. ISSN 0020-0255.

prof. Ing. Jiří Fotr, CSc.
University of Economics, Prague
Faculty of Business Administration
jiri.fotr@vse.cz

Ing. Lenka Švecová, Ph.D.
University of Economics, Prague
Faculty of Business Administration
lenka.svecova@vse.cz

doc. Dr. Ing. Miroslav Plevný
University of West Bohemia in Pilsen
Faculty of Economics
plevny@kem.zcu.cz

doc. Ing. Emil Vacík, Ph.D.
University of West Bohemia in Pilsen
Faculty of Economics
vacik@kpm.zcu.cz

Doručeno redakci: 29. 3. 2013
Recenzováno: 22. 5. 2013, 30. 5. 2013
Schváleno k publikování: 27. 9. 2013

Abstract

MULTI-CRITERIA PROJECT PORTFOLIO OPTIMIZATION UNDER RISK AND SPECIFIC LIMITATIONS

Jiří Fotr, Miroslav Plevný, Lenka Švecová, Emil Vacík

The development of a portfolio of investment projects is a relatively underestimated economic practice, which often leads to wrong investment decisions with a negative impact on the corporate performance. This development is often done under certainty, which means with the only one possible scenario. The multi-criteria nature of the task character is also rarely respected. The evaluation of projects is usually done in isolation, without any connection to other projects or without taking into account dependencies between the projects.

This paper aims to specify the problem of optimization of development of a project portfolio under risk (optimal allocation of scarce resources). The article offers two approaches to the optimization. The first approach is based on deterministic equivalents with application of bivalent programming (multi-criteria and mono-criteria optimization). The second one uses stochastic optimization based on the Monte Carlo simulation. The application of these model approaches can greatly improve the quality of the project portfolio development under risk.

Key Words: *Project portfolio development, simulation Monte Carlo, investment projects, risk.*

JEL Classification: *C15, C61, D81, L25, M21.*